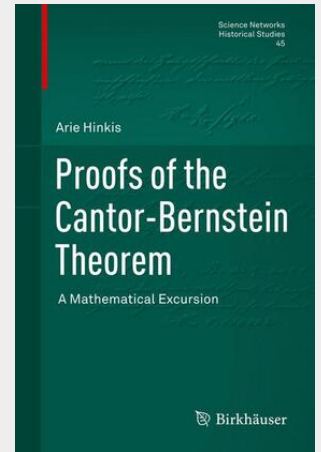Hinkis

Proofs of the Cantor-Bernstein Theorem

A Mathematical Excursion

This book offers an excursion through the developmental area of research mathematics. It presents some 40 papers, published between the 1870s and the 1970s, on proofs of the Cantor-Bernstein theorem and the related Bernstein division theorem. While the emphasis is placed on providing accurate proofs, similar to the originals, the discussion is broadened to include aspects that pertain to the methodology of the development of mathematics and to the philosophy of mathematics. Works of prominent mathematicians and logicians are reviewed, including Cantor, Dedekind, Schröder, Bernstein, Borel, Zermelo, Poincaré, Russell, Peano, the Königs, Hausdorff, Sierpinski, Tarski, Banach, Brouwer and several others mainly of the Polish and the Dutch schools. In its attempt to present a diachronic narrative of one mathematical topic, the book resembles Lakatos' celebrated book Proofs and Refutations. Indeed, some of the observations made by Lakatos are corroborated herein. The analogy between the two books is clearly anything but superficial, as the present book also offers new theoretical insights into the methodology of the development of mathematics (proof-processing), with implications for the historiography of mathematics.

The chief purpose of the book is to present, in detail, a compilation of proofs of the Cantor-Bernstein Theorem (CBT) published through the years since the 1870's. Over thirty such proofs are surveyed. The book comprises five parts. In the first part the discussion covers the role of CBT and related notions in the writings of Cantor and Dedekind. New views are presented, especially regarding the general proof of CBT obtained by Cantor, his proof of the Comparability Theorem, the ruptures in the Cantor-Dedekind correspondence and the origin of Dedekind's proof of CBT. The second part covers the first CBT proofs published (1896-1901). The works of the following mathematicians is considered in detail: Schröder, Bernstein, Bore, Schoenflies and Zermelo. Here a subtheme of the book is launched; it concerns the research project following Bernstein's Division Theorem (BDT). In its third part the book covers proofs that emerged during the period when the logicist movement was developed (1902-1912). It covers the works of Russell and Whitehead, Jourdain, Harward, Poincaré, J. König, D. König (his results in graph theory), Peano, Zermelo, Korselt. Also Hausdorff's paradox is discussed linking it to BDT. In the fourth part of the book are discussed the developments of CBT and BDT (including the inequality-BDT) in the hands of the mathematicians of the Polish School of Logic, including Sierpinski, Banach, Tarski, Lindenbaum, Kuratowski, Sikorski, Knaster, the British Whittaker, and Reichbach. Finally, in the fifth part, the main discussion concentrates on the attempts to port CBT to intuitionist mathematics (with results by Brouwer, Myhill, van Dalen and Troelstra) and to Category Theory (by Trnková and Koubek). The second purpose of the book is to develop a methodology for the comparison of proofs. The core idea of this methodology is that a proof can be described by two descriptors, called gestalt and metaphor. It is by comparison of their descriptors that the comparison of proofs is obtained. The process by which proof descriptors are extracted from a proof is named 'proof-processing', and it is conjectured that mathematicians perform proof-processing habitually, in the study of proofs.

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